Investigating Mixed Memory-Reinforcement Models for Random Walks

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Random Walks in Biology

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- Examples
  - Path of a molecule in a gas
  - Motion of a slime mold towards food
  - Movement of ants between food source and anthill
- Not necessarily purely random

Memory and Reinforcement

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- **Reinforcement**: higher probability of moving along previous paths taken by other particles
  - Ants following trails of chemical pheromone
  - Causes slower spread of particles away from starting location

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- Build a model in which memory and reinforcement are both factors
  - More realistic biologically
  - Possible optimum memory-reinforcement mix for least travel time
  - Reproduce and explain phenomena such as death spiral

Memory Models

Memory Models

- Memory involves angles θ of deflection
- Memory parameter m

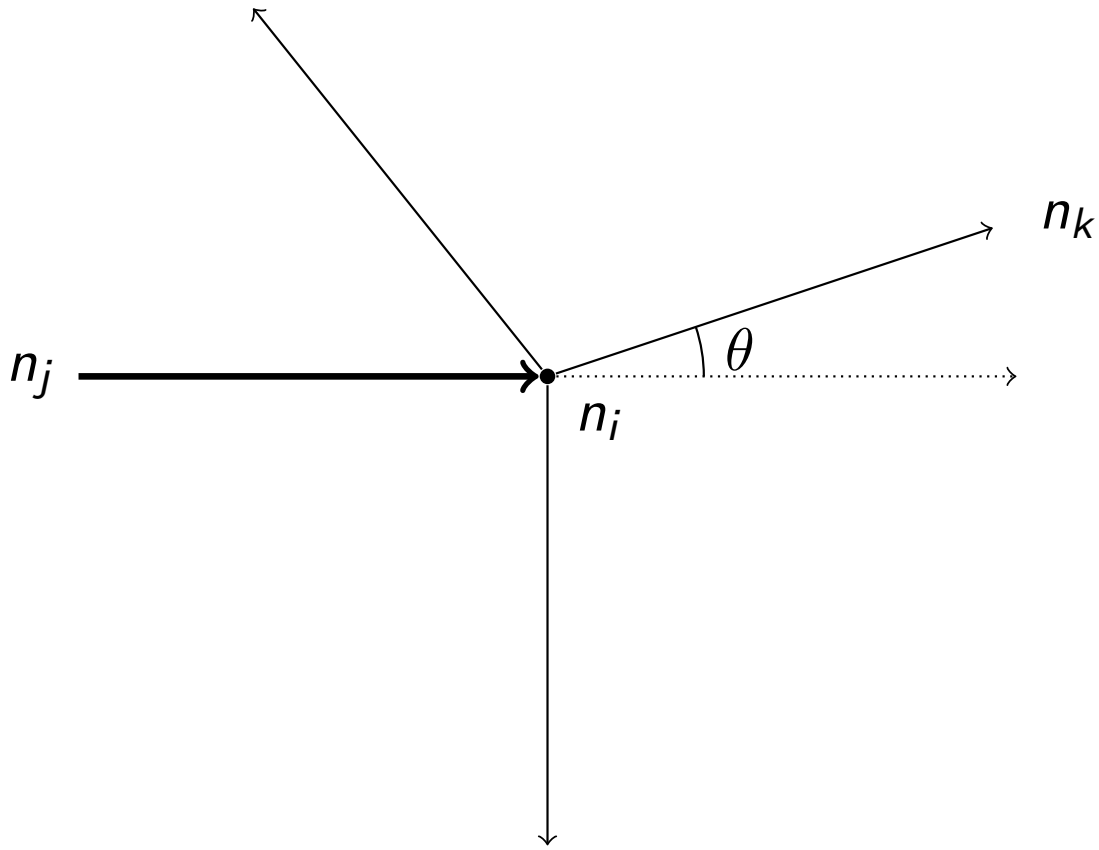
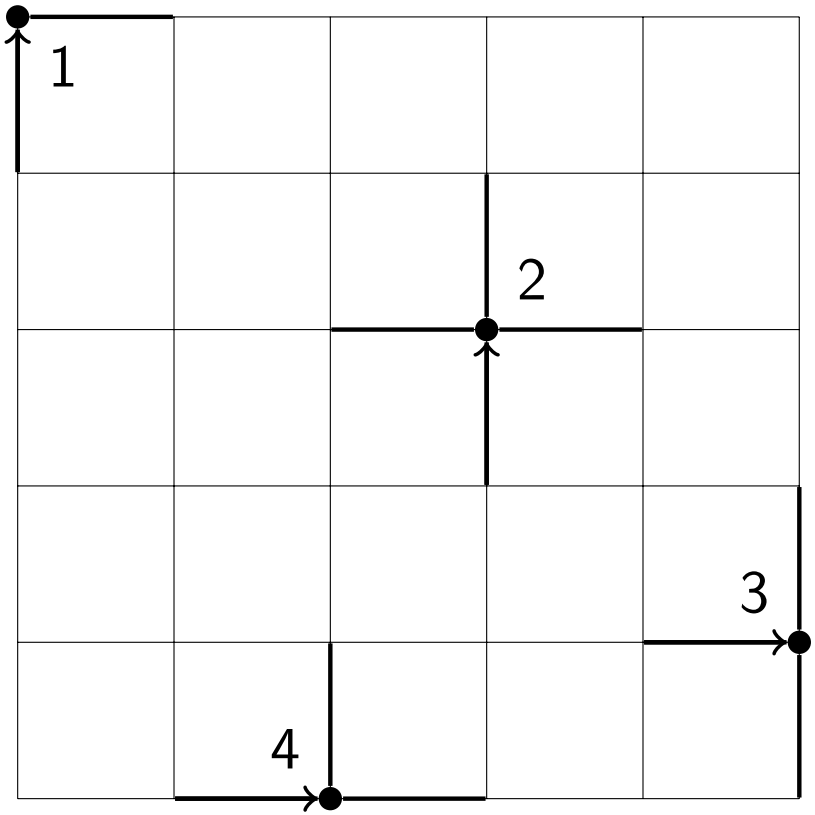


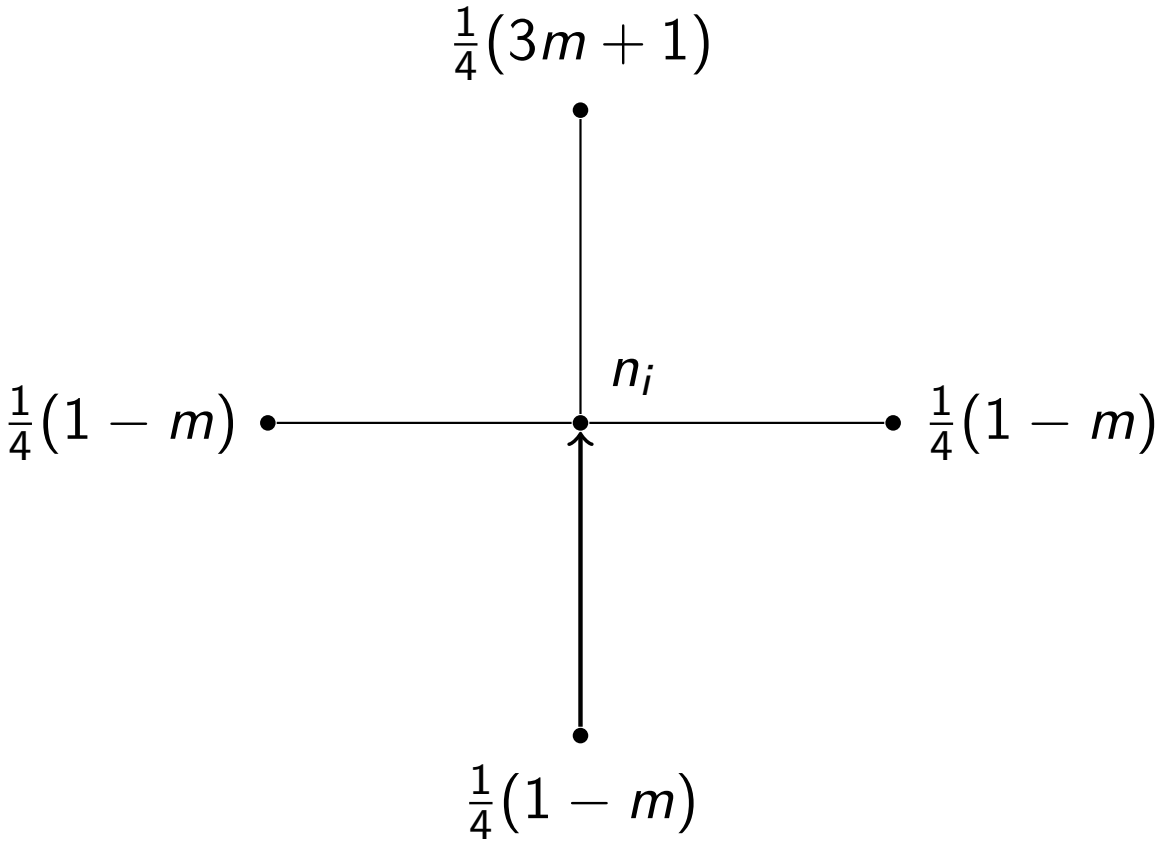
Figure: General intersection in a graph

Memory Models

- Simple model: Rectangular Grid



Memory Models



Memory Models

- General graph

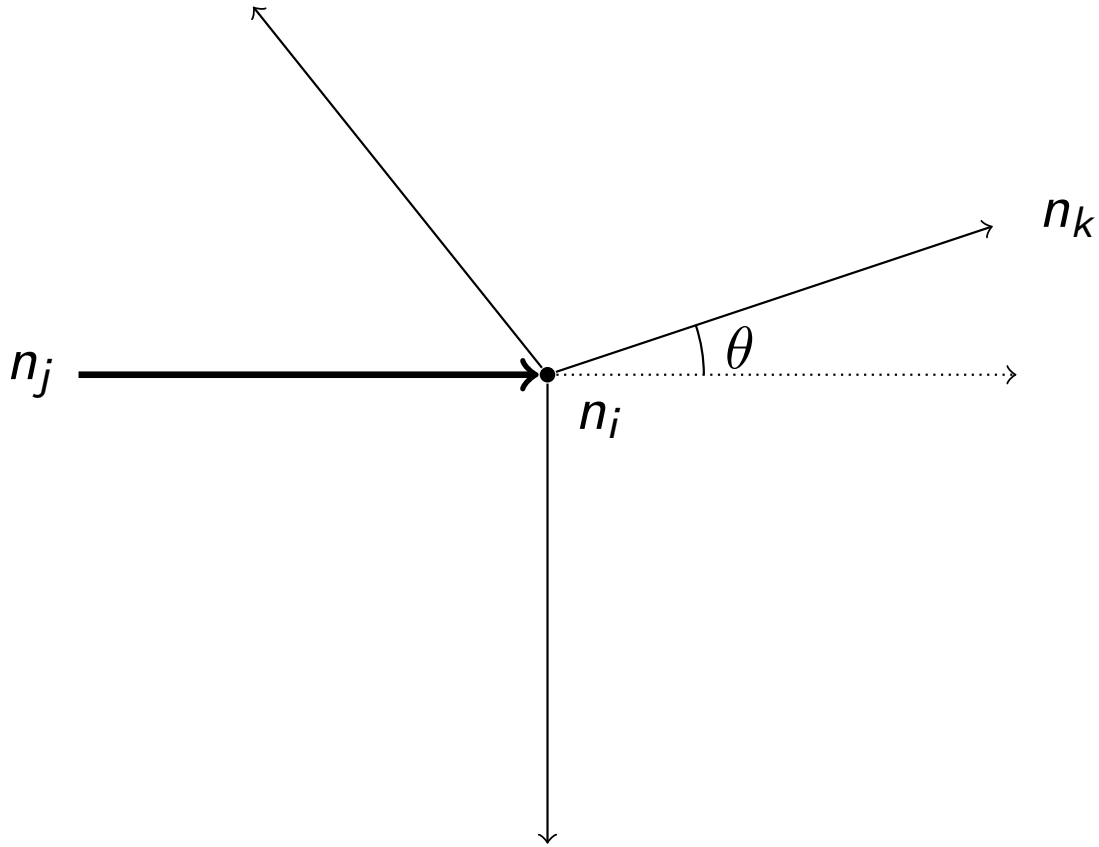
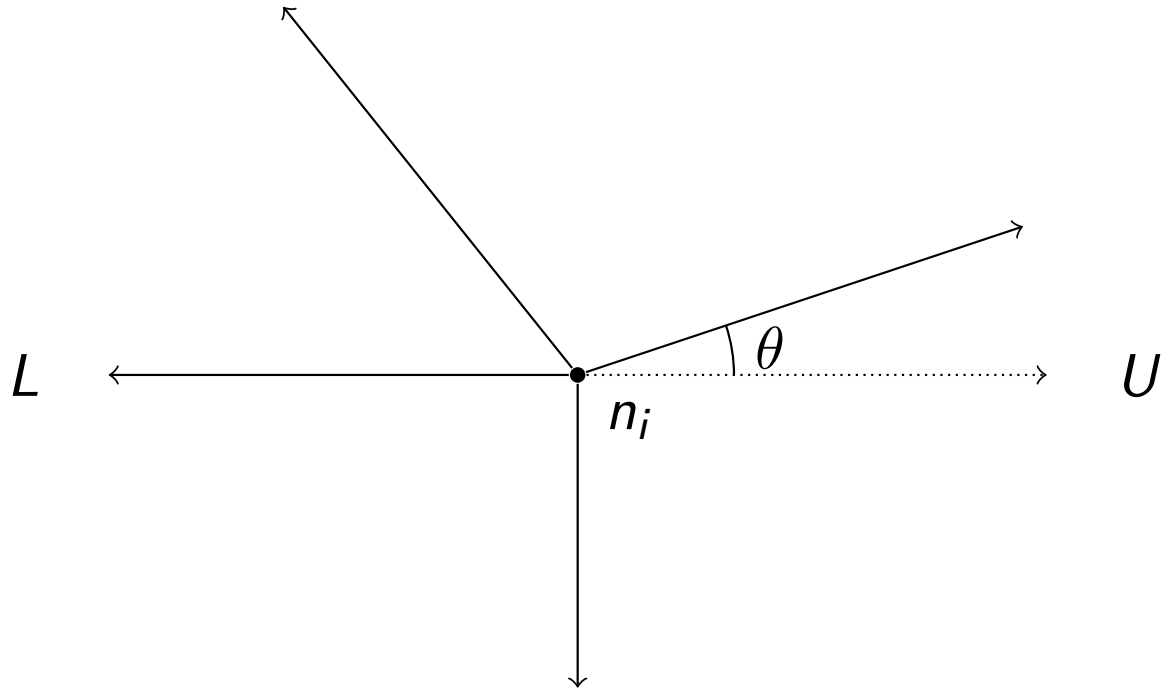


Figure: General intersection

Memory Models

- Assign U and L weights to forward and backward directions



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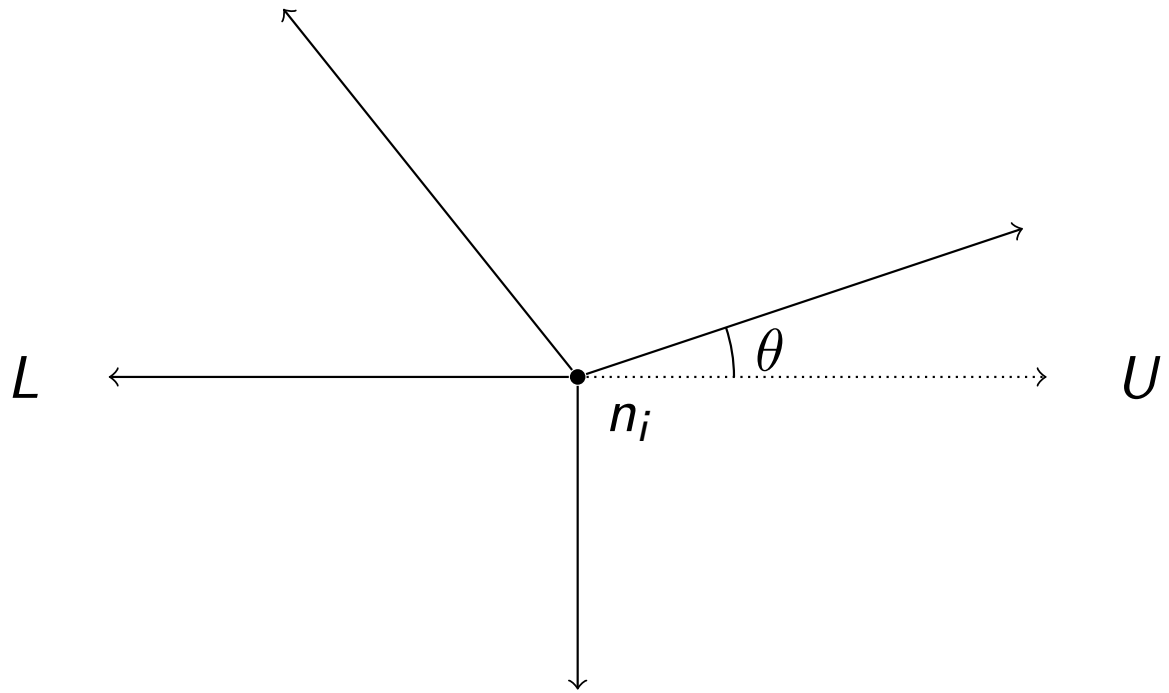

Figure: General intersection

$$W(\theta) = (U - L)f(\theta) + L$$

Memory Models

$$f(\theta) = \frac{e^{\frac{\frac{U-1}{\frac{1}{2}-U}(\pi-\theta)}} - 1}{e^{\frac{\frac{U-1}{\frac{1}{2}-U}\pi}} - 1}$$

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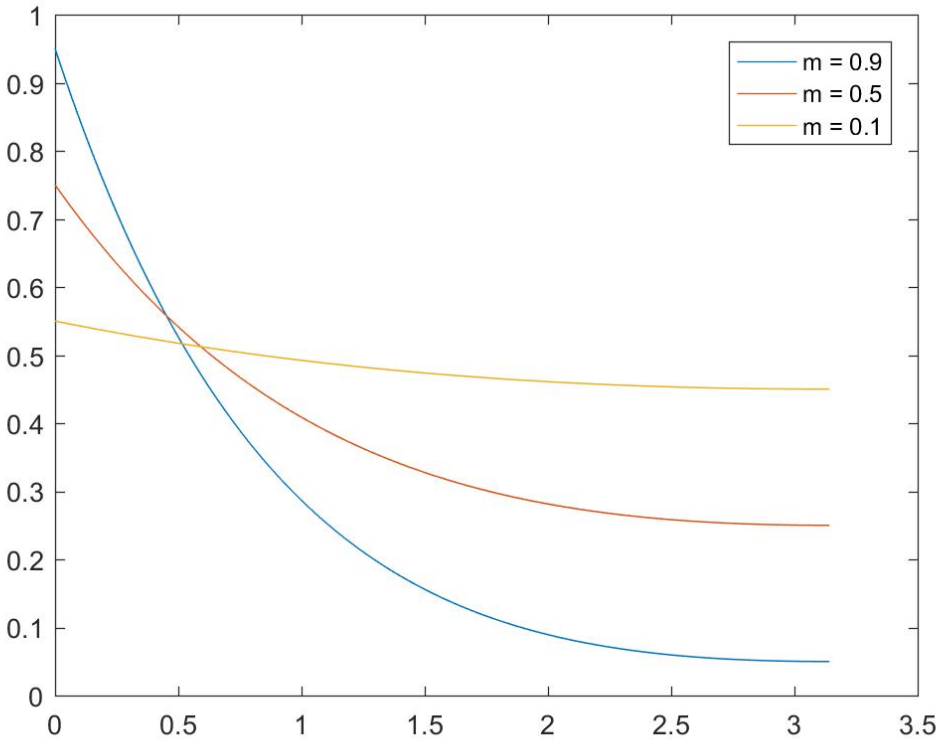

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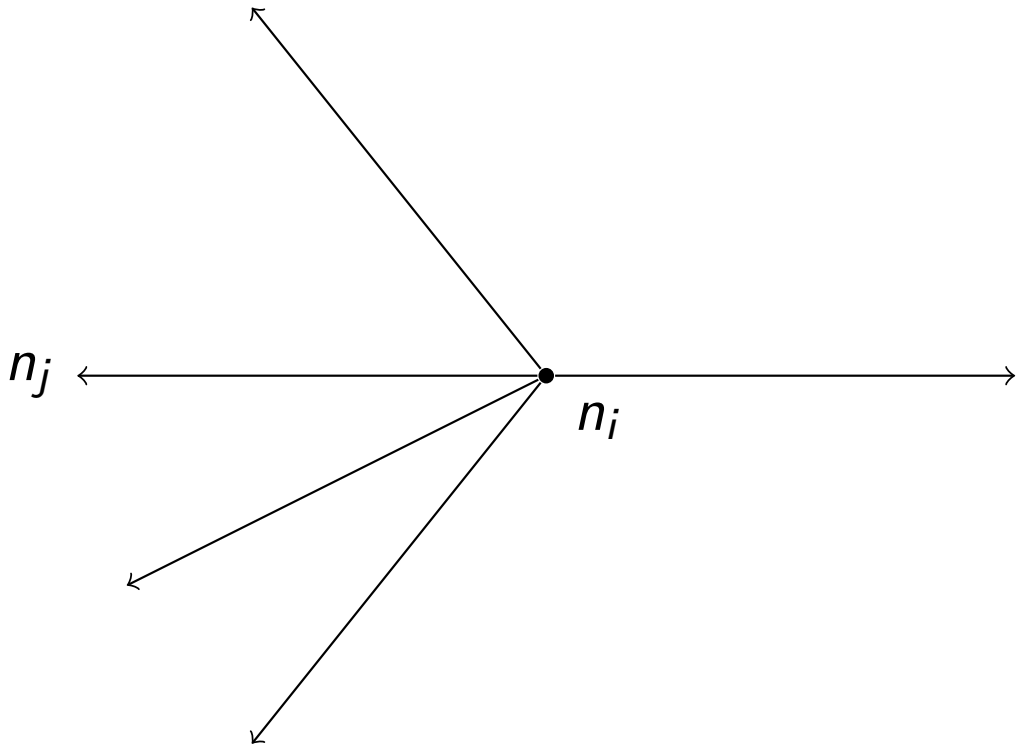

Reinforced Random Walks

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- Probability depends on pheromone concentration (d_{ij}) and edge length (l_{ij})

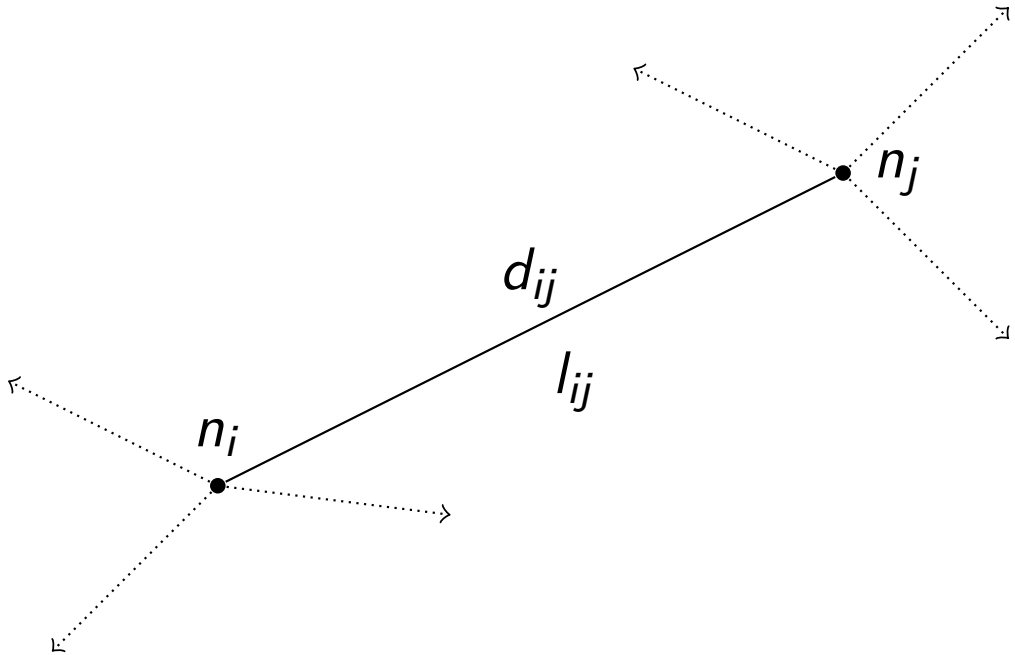

Figure: General edge E_{ij}

Reinforced Random Walks

- Mean flow rate ($\bar{I}_{ij}$) equation based on edge weights (Ma Q, et. al.):

$$\bar{I}_{ij} = \left(\frac{N_i}{\sum_{e \in E_i} \frac{d_e}{l_e}} - \frac{N_j}{\sum_{e \in E_j} \frac{d_e}{l_e}} \right) \left(\frac{d_{ij}}{l_{ij}} \right)$$

- N_i is the number of particles at node n_i
- E_i is the set of all edges around node n_i
- d_e is the pheromone concentration on an edge e
- l_e is the length of an edge e

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- Reinforcement weight is $1 - m$

Results

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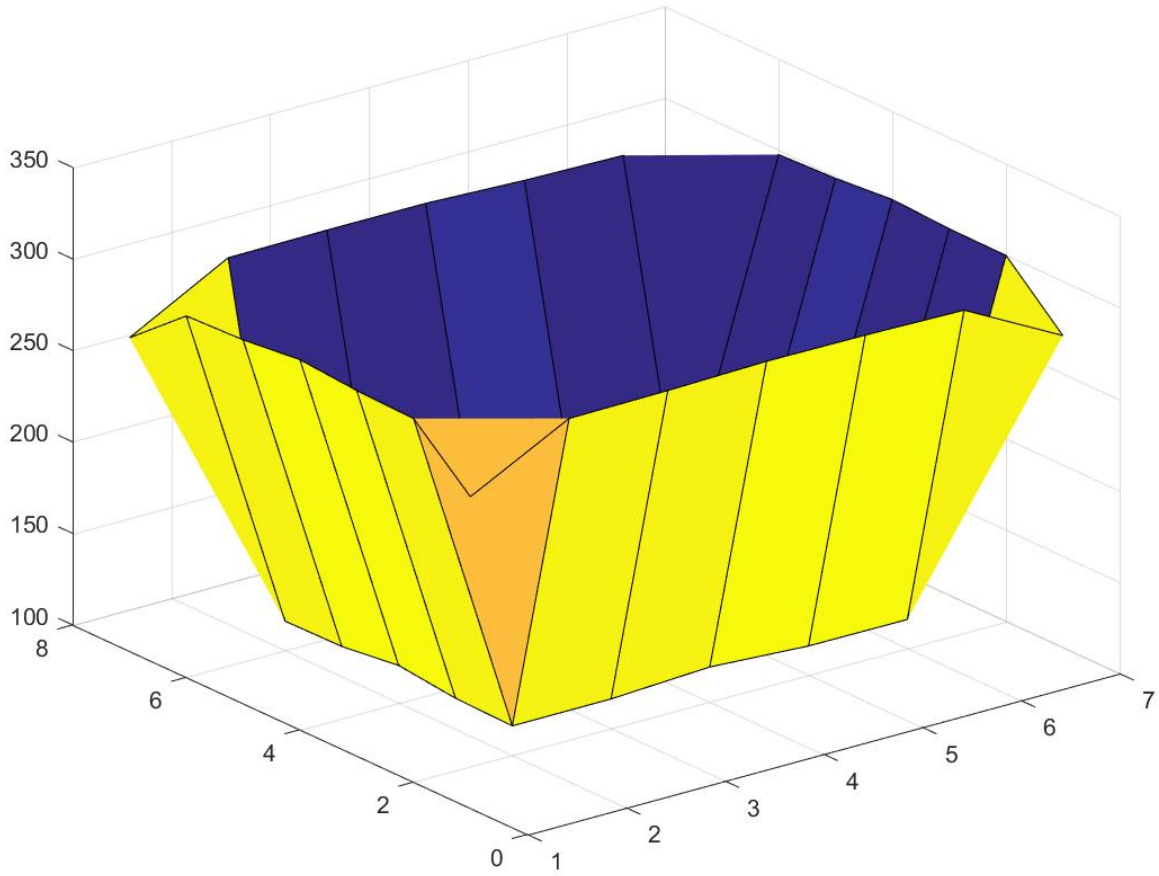
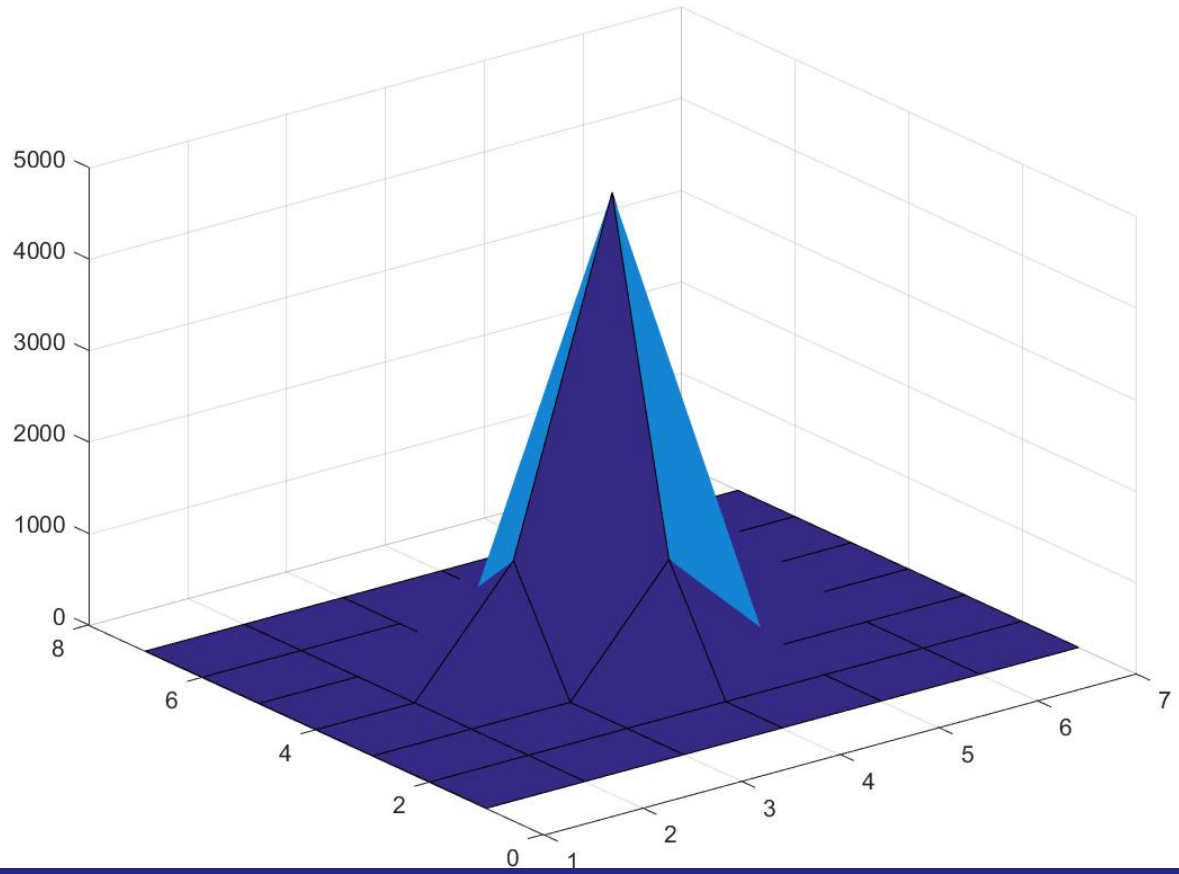
- **Pure memory** on 7 x 7 grid: memory=1.0, reinforcement=1 evaporation=0.01, reflection=0.1, times=1000

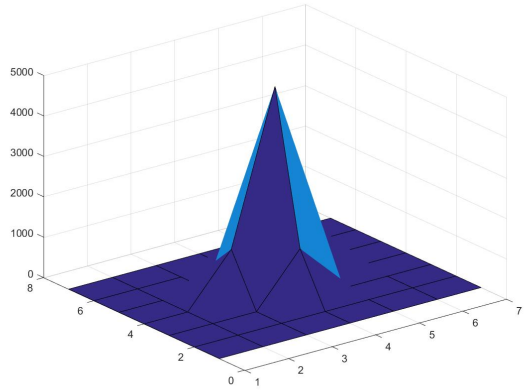


Results

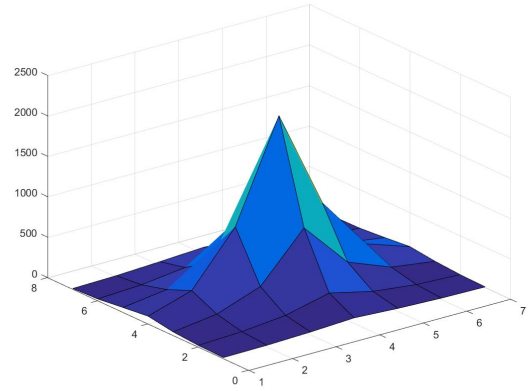
- **Pure reinforcement** on 7 x 7 grid: memory=0, reinforcement=1,evaporation=0.01, reflection=0.1, times=1000



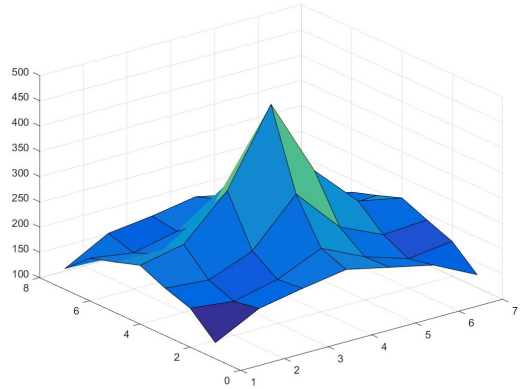
Results: Mixed Memory Reinforcement Model



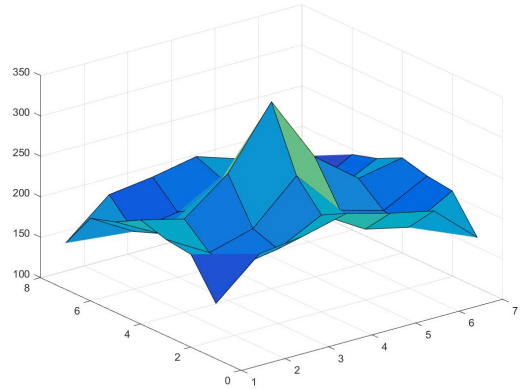
(a) $m = 0$



(b) $m = 0.1$



(c) $m = 0.3$



(d) $m = 0.4$

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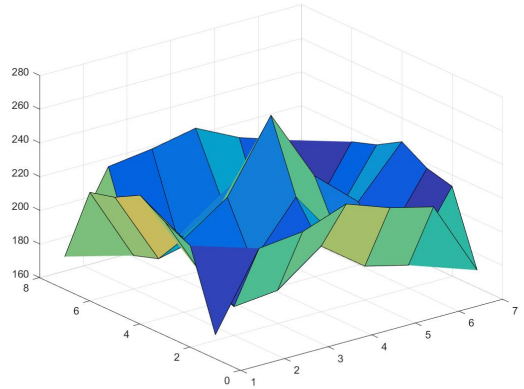
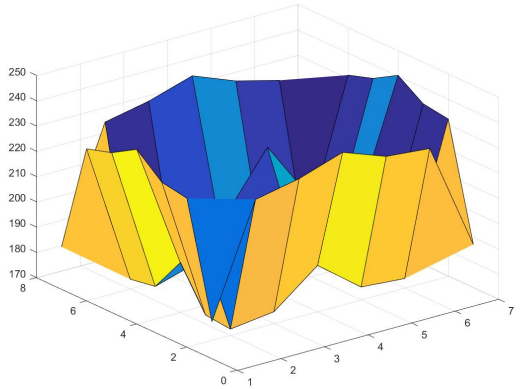
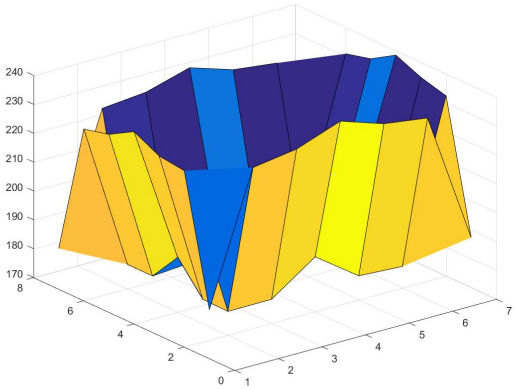


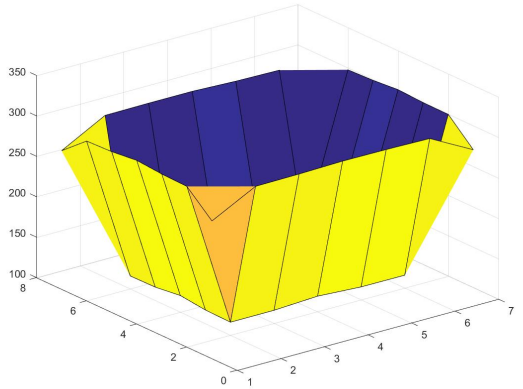
(a) $m = 0.5$



(b) $m = 0.6$

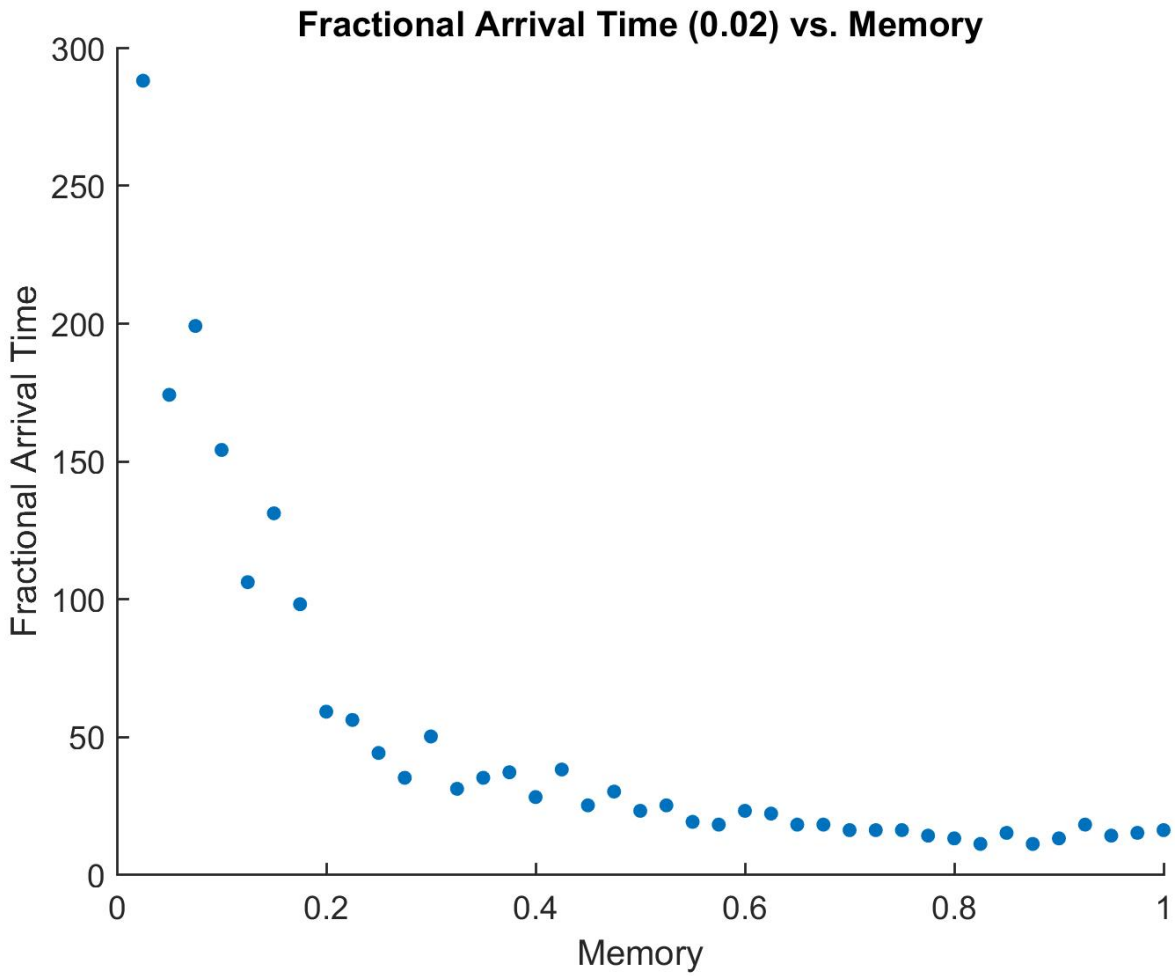


(c) $m = 0.7$



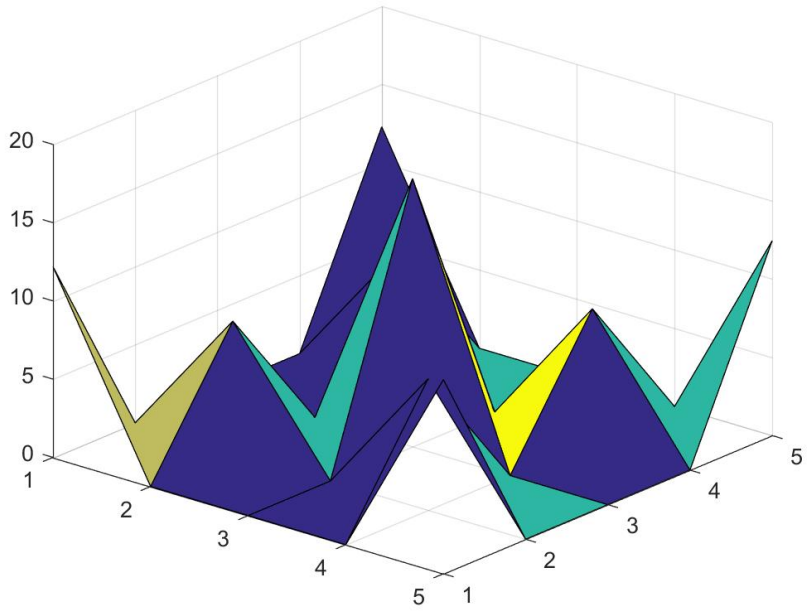
(d) $m = 1$

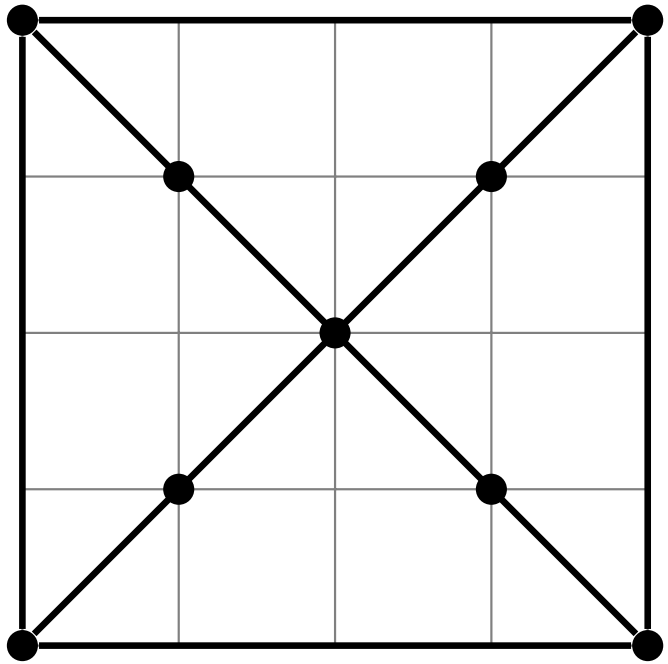
Shortest Arrival Time



Generalized Graph Model

- Early results





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- Reproducing the death spiral

Acknowledgements

Thanks to

- My mentor Andrew Rzeznik
- My parents
- and the MIT-PRIMES program.